Notes: Put on bright yellow paper!

"Derive" the Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$\sin^2\theta$ $\qquad$ $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = \csc^2\theta$$

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$$\tan^2\theta + 1 = \sec^2\theta$$

Derive the other two identities using the given identity. Show work separate from the identities you previously wrote.

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$$\tan^2\theta + 1 = \sec^2\theta$$

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Hint: sometimes you will want to rearrange the identities for substitution

$$\sin^2\theta + \cos^2\theta = 1$$

$$-\sin^2\theta \qquad -\sin^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

↑ equivalent to the original equation

$$1 + \cot^2\theta = \csc^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

Identities Practice Sheet #1

Rewrite each expression by factoring.

1. $\cos x - \cos x \cdot \tan^2 x =$ $\cos x (1 - \tan^2 x)$

2. $\csc^2 x \cdot \cot^2 x + \csc^2 x =$ $\csc^2 x (\cot^2 x + 1)$

3. $\sec^2 x - 1 =$ $(\sec x + 1)(\sec x - 1)$

4. $\sin^2 x - \cos^2 x = (\sin x + \cos x)(\sin x - \cos x)$

5. $2\cos^2 x - \cos x = \cos x(2\cos x - 1)$

6. $1 - \sin^2 x = (1 + \sin x)(1 - \sin x)$

7. $\sec^2 x - \tan^2 x$ $(\sec x + \tan x)(\sec x - \tan x)$

8. $\sin x + \sin x \cos x = \sin x(1 + \cos x)$

Simplify each expression. Clearly show all steps.

9. $\cos x + \cos x \cdot \tan^2 x$

10. $\sin^2 x \cdot \cot^2 x + \sin^2 x$

$\cos x (1 + \tan^2 x)$

$= \cos x (\sec^2 x)$

$= \frac{\cos x}{1} \left(\frac{1}{\cos^2 x} \right)$

$= \frac{1}{\cos x} = \sec x$

Hints:

9. factor common term, substitute Pythagorean identity, rewrite using reciprocal identity
10. factor common term, substitute Pythagorean identity, rewrite using reciprocal identity
11. factor difference of squares, then cancel like terms
12. factor difference of squares, then cancel like terms
13. substitute Pythagorean identity, then rewrite using reciprocal identity
14. substitute Pythagorean identities, rewrite using reciprocal identity, use quotient identity
15. factor the numerator and the denominator, cancel like terms, split fraction apart

Verify that each equation is an identity.

rewrite/simplify

keep as is

14. $(\tan^2 x + 1)(1 - \cos^2 x) = \tan^2 x$

$(\sec^2 x)(\sin^2 x) = \tan^2 x$

$\frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1} = \tan^2 x$

$\frac{\sin^2 x}{\cos^2 x} = \tan^2 x$

$\tan^2 x = \tan^2 x$ ✓

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$